Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2019

Question/Answer booklet

MATHEMATICS APPLICATIONS

Section One: Calculator-free

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

WA student number: In figures

In words

Number of additional answer booklets used (if applicable):

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	55	35
Section Two: Calculator-assumed	11	11	100	100	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (55 Marks)**

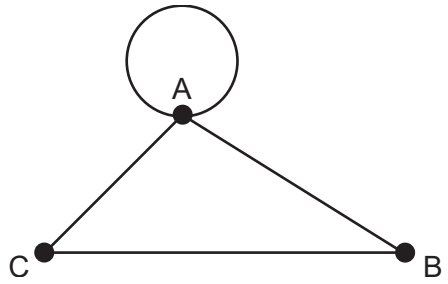
This section has **six** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1 **(8 marks)**

The graph shown represents three buildings A, B and C, with connecting walkways, at a local school.



(a) Why is the graph planar? (1 mark)

(b) Show that the graph satisfies Euler's formula. (2 marks)

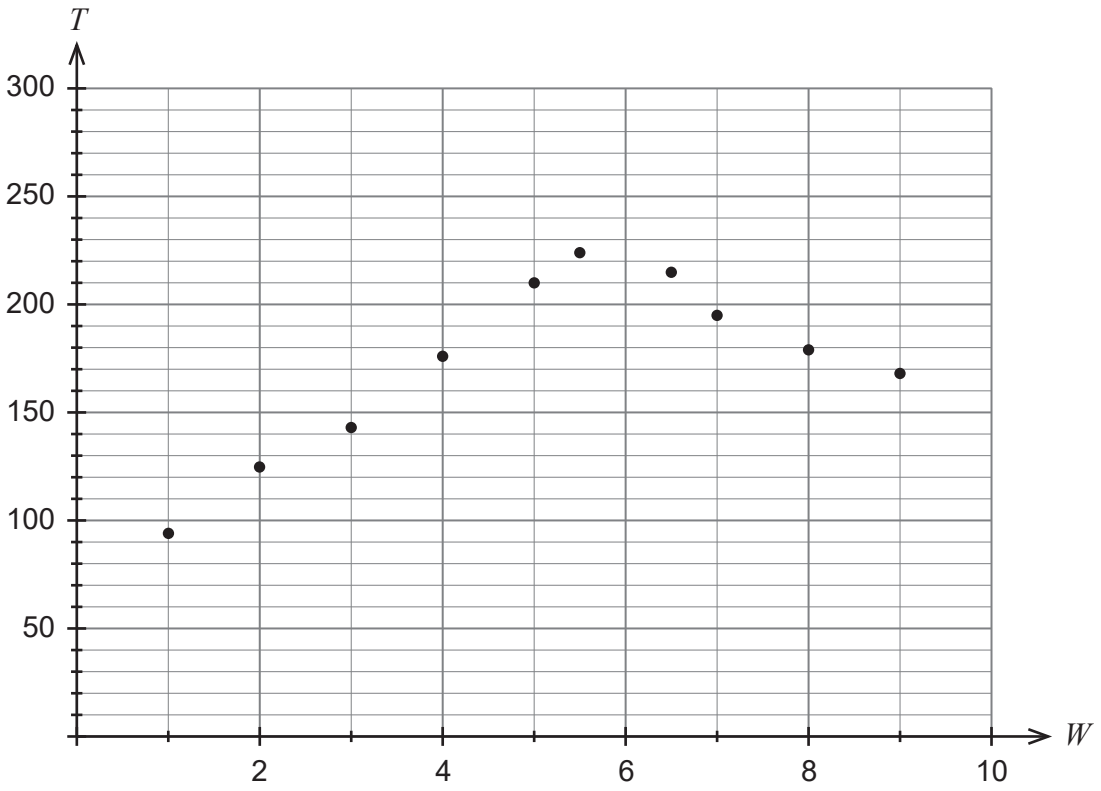
(c) Construct the adjacency matrix for the graph. (3 marks)

A student wishes to carry out closed walks of length two from Building A.

(d) List all his possible walks. (2 marks)

Question 2 **(9 marks)**

Katie is a hobby farmer who has been experimenting with a species of tomato plant growing under the same soil and climatic conditions. She varied the amount of water (W), in millimetres, used during each week and recorded the total number of tomatoes (T) produced by each plant. The scatterplot showing her results is drawn below.



Katie determined the following summary information:

- $r_{WT} = 0.66$
- the equation of the least-squares line is $T = 10.55W + 119.11$

(a) Identify the response variable. (1 mark)

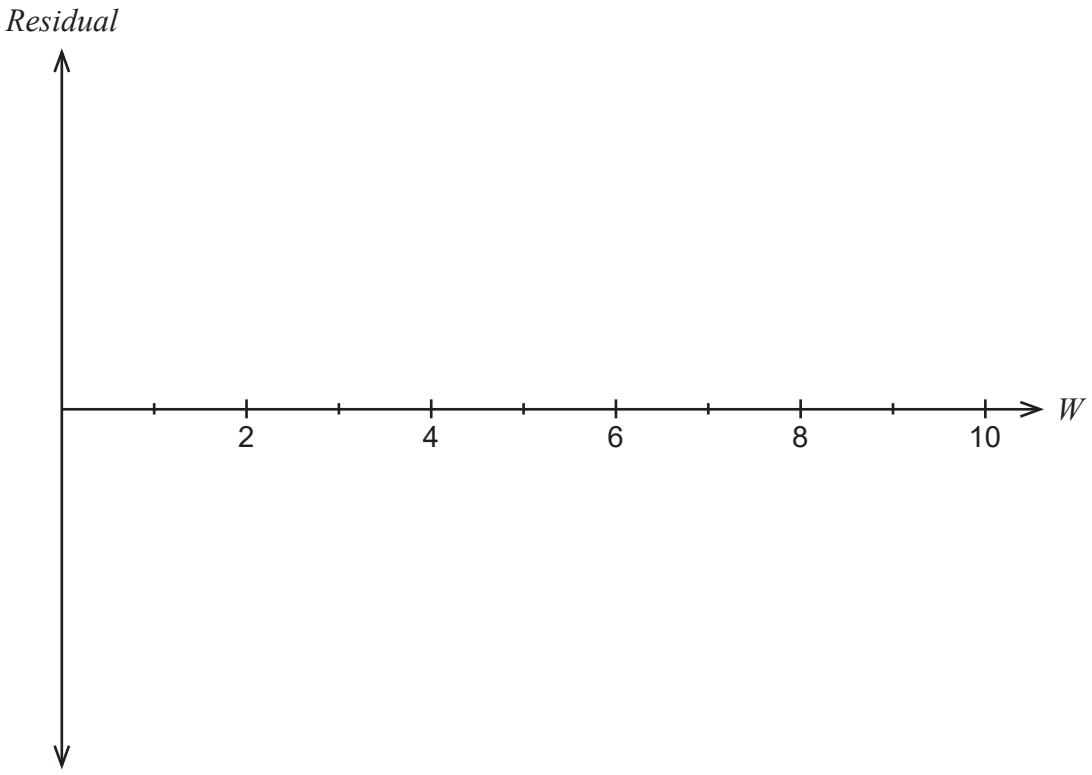
(b) Use the equation of the least-squares line to predict the total number of tomatoes produced when 10 millimetres of water are given to a plant during each week. (2 marks)

See next page

(c) Fit the least-squares line to the scatterplot. (2 marks)

Katie decided to draw a residual plot to gather more information about her results.

(d) (i) Sketch a residual plot she would have likely drawn for the given data.
Note: you do not have to calculate actual values. (2 marks)



(ii) Use your residual plot to discuss the appropriateness of fitting a linear model to the data. (2 marks)

Question 3 **(10 marks)**

A company has four small workshops that each produce four different types of outdoor furniture. The annual cost of production of the furniture at each workshop is shown in the table below, with all values in thousands of dollars.

	Type 1 $'000	**Type 2 $'000**	**Type 3 $'000**	**Type 4 $'000**
Workshop A	25	43	50	39
Workshop B	33	31	56	39
Workshop C	28	47	59	38
Workshop D	36	32	56	41

The cost matrix is given by

$$\begin{bmatrix} 25 & 43 & 50 & 39 \\ 33 & 31 & 56 & 39 \\ 28 & 47 & 59 & 38 \\ 36 & 32 & 56 & 41 \end{bmatrix}$$

The company is interested in knowing what the minimum annual cost would be if each furniture type was allocated to its own individual workshop. The Hungarian Algorithm is to be used to determine the allocation and the minimum annual cost. The first step of the Hungarian Algorithm, where the smallest number in each row is subtracted from all other numbers in that row, is shown below.

(a) Continue the steps of the Hungarian Algorithm to determine the appropriate allocation of workshops to furniture type and state the **minimum** annual cost. (5 marks)

$$\begin{bmatrix} 0 & 18 & 25 & 14 \\ 2 & 0 & 25 & 8 \\ 0 & 19 & 31 & 10 \\ 4 & 0 & 24 & 9 \end{bmatrix}$$

Type	Type 1	Type 2	Type 3	Type 4
Workshop				

Total minimum annual cost _____

See next page

The revenue matrix, in thousands of dollars, for the sale of the furniture produced annually at each workshop is given by

$$\begin{bmatrix} 37 & 61 & 60 & 53 \\ 45 & 52 & 73 & 50 \\ 38 & 65 & 75 & 55 \\ 44 & 54 & 76 & 45 \end{bmatrix}$$

(b) Given that $Profit = Revenue - Cost$, complete the Profit matrix below. (1 mark)

$$\text{Profit matrix} = \begin{bmatrix} 12 & 18 & 10 & 14 \\ 12 & 21 & 17 & 11 \\ 10 & 18 & & \\ & & & \end{bmatrix}$$

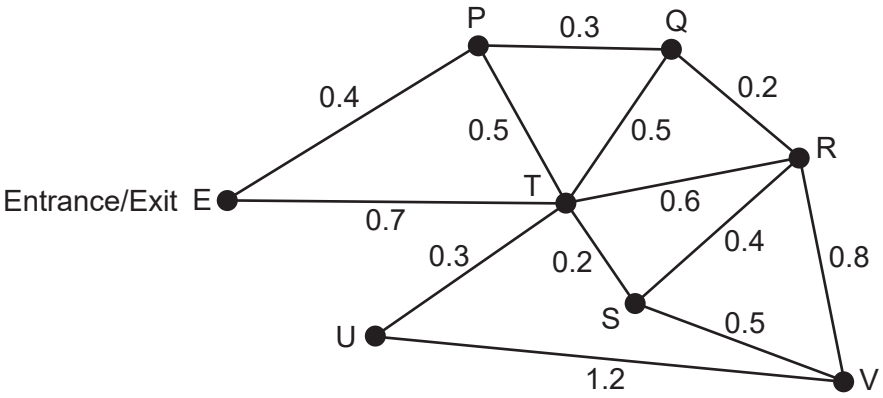
(c) Use the Hungarian Algorithm to determine the appropriate allocation of workshops to furniture type that will produce the **maximum** annual profit. (4 marks)

Type	Type 1	Type 2	Type 3	Type 4
Workshop				

See next page

Question 4 **(11 marks)**

A marine park has attractions with paths connecting them. The vertices on the graph represent the attractions and the numbers on the edges represent the path distances (km) between the attractions. Visitors can either walk around the park or take one of the many shuttle buses that run between attractions.



The manager of the marine park leaves his office, which is located at the entrance/exit (E) and walks to attraction V.

(a) (i) Determine the shortest distance from E to V. (1 mark)

(ii) If the manager needs to pick up some tools left at U on the way, determine the route he should take and the corresponding distance, given he wants to take the shortest route from E to V. (2 marks)

Rachel arrives at the entrance. She wants to complete a Hamiltonian cycle.

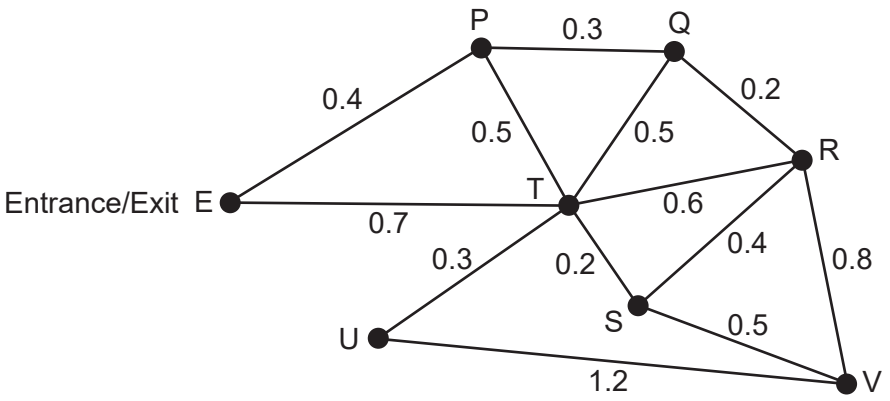
(b) State the route she should take. (2 marks)

See next page

Drinking water is already being supplied at E. The manager has recently received funding to establish drinking fountains at each attraction. For this to happen, pipelines will need to be laid along the paths to each attraction. He has drawn up a table to show the distances between attractions.

	E	P	Q	R	S	T	U	V
E	–	0.4	–	–	–	0.7	–	–
P	0.4	–	0.3	–	–	0.5	–	–
Q	–	0.3	–	0.2	–	0.5	–	–
R	–	–	0.2	–	0.4	0.6	–	0.8
S	–	–	–	0.4	–	0.2	–	0.5
T	0.7	0.5	0.5	0.6	0.2	–	0.3	–
U	–	–	–	–	–	0.3	–	1.2
V	–	–	–	0.8	0.5	–	1.2	–

(c) (i) Use Prim's algorithm, or otherwise, to determine the minimum total length of pipelines. Highlight the required pipelines on the diagram below. (4 marks)

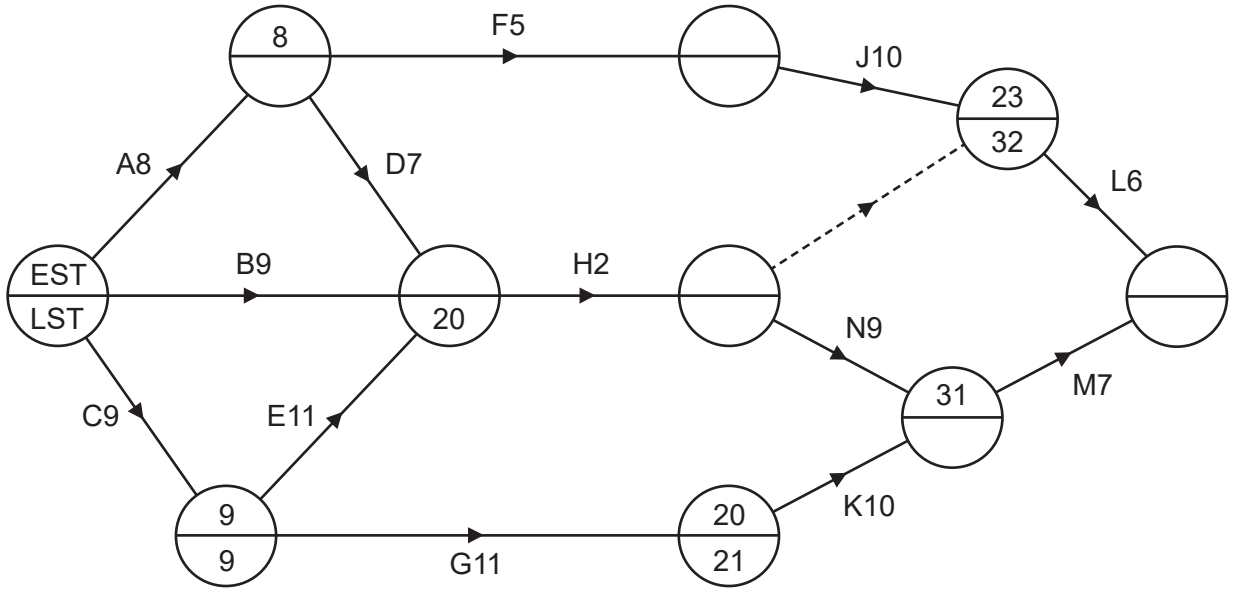


(ii) The manager has been told that a pipeline of length 0.2 km could be laid from S to U. How, if at all, will this affect the total length of pipelines that should be laid in order to maintain a minimum length? (2 marks)

See next page

Question 5 **(11 marks)**

The network below represents a construction project. The number on each edge gives the time, in hours, to complete the activity. Each activity requires one worker.



(a) Complete the precedence table below. (2 marks)

Activity	A	B	C	D	E	F	G	H	J	K	L	M	N
Time (hours)	8	9	9	7	11	5	11	2	10	10	6	7	9
Immediate predecessor	-	-	-	A	C	A	C	B,D,E					

(b) Complete the network showing the earliest starting time (EST) and latest starting time (LST) for each node. (Note: the first node indicates which is the EST and the LST.) (2 marks)

See next page

(c) Determine the critical path and the minimum completion time for the project. (2 marks)

(d) Calculate the float times for Activities D and F. (2 marks)

(e) Given that the sum of all the times of the activities is 104 hours, calculate the minimum number of workers required to complete the project in the minimum completion time. (1 mark)

(f) What is the latest time into the project that Activity F could start without affecting the minimum completion time? (1 mark)

(g) Explain the purpose of the dotted line on the network. (1 mark)

See next page

Question 6 **(6 marks)**

The population of turtles in an artificial lake at a wildlife sanctuary is initially 32 and research has shown a natural decrease in population of 50% each year. Twenty extra turtles are introduced to the lake at the end of each year.

(a) Determine a recursive rule for the turtle population. (2 marks)

(b) Determine the long-term steady state of the turtle population. (2 marks)

(c) If the wildlife sanctuary preferred a long-term steady state of 80 turtles, what yearly addition of turtles would be required to produce this steady state? Assume all other conditions remain the same. (2 marks)

End of section

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

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